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# THE DIMENSIONS OF THE STARS[1]

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It is very natural for the layman, seeing the title of this lecture, to ask why we should wish particularly to know the dimensions of the stars, what purpose a knowledge of the sizes, masses and densities of the stars will serve. Before attempting to describe how they are determined, it has seemed desirable, in order to give, as it were, a plot to the story and to add interest to a dull array of figures, to answer why it is necessary to determine the sizes and masses of the stars. Moreover, the answer to this question forms one of the most interesting episodes in the recent history of astronomy, while also the methods by which the dimensions of these tremendously remote bodies are determined form most interesting examples of the application of scientific methods to a difficult and apparently almost insoluble problem.

A knowledge of the dimensions of the stars is of course a necessary part in the main astronomical problem, the determination of the constitution of the universe: for in addition to the knowledge of the positions and motions and of the physical and chemical constitution of the stars, we should evidently also know their size and mass. Apart, however, from the general interest attached to and the value of this knowledge for the main problem, definite information about the dimensions of the stars has a special interest in connection with the theory of stellar evolution. The human mind is never satisfied with a mere collection of facts or observations. It must attempt to coördinate and explain such facts and observations. Thus astronomers observed that in the heavens there were stars of different brightness and color, whose spectra could be arranged in a continuous sequence, seeming to progress naturally from one type to another. When they also observed that there was a different class of heavenly bodies,

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the nebulæ, distinguished by diffuseness and size from the stars, and yet with similarities in their spectra, it was inevitable that they should imagine that the nebulæ and the different types of stars represented only different stages of development and they should attempt to link them together and to trace the process of evolution.

The theory almost universally accepted until perhaps five years ago assumed that the nebulæ, enormously extended collections of very tenuous gaseous or meteoric matter, formed the primal substance which by condensation developed into stars. The first stage in the process resulted in the very hot blue stars of which those in *Orion* with *Vega* and *Sirius* are examples, stars at temperatures between 20,000° C. and about 11,000° C. Such stars were much less dense than the Sun and radiated energy into space at an enormous rate. The spectra of these stars, which are designated by the letters O, B, A, are comparatively simple, containing lines due mostly to the light gases hydrogen and helium and other non-metallic elements. The condensation and cooling continued passing through the white stage, temperatures 10,000° to 8,000°, the spectra now being of the F-type in which metallic lines become prominent. The next stage is the G-type stars like the Sun at temperatures of about 6,000°, passing to K-type yellow stars like *Arcturus* at 4,500°, to the red M and N type banded spectra stars at temperatures of 3,000° or less, and then to extinction. The theory required continuously decreasing temperature, decreasing diameter, and increasing density of the stars throughout the whole range. Although energy was continuously supplied by the contraction, it was radiated out into space more rapidly with consequent lowering of temperature.

The more recent theory of the mode of development of the stars, though commencing with the primal nebula as before, postulates on the contrary both an increasing and decreasing temperature scale. Such a theory was propounded by Sir Norman Lockyer about thirty years ago from the spectral characteristics of the stars but, owing to inherent difficulties, never received general acceptance. The new and now generally accepted theory of stellar evolution was developed by Prof. H. N.

Russell of Princeton, who, practically single-handed and in spite of the prevailing belief in the older hypothesis and the natural conservatism of astronomers, has convincingly demonstrated the superiority of the newer idea of evolution. According to Russell's theory, the inter-gravitational attraction of the particles of the tenuous extended primal nebula, which is probably at a low temperature, causes condensation and consequent increase of temperature. A familiar example of a similar method of producing heat is given when air is pumped into a bicycle or automobile tire. The air becomes hot and heats the pump not because of friction but on account of the work done in forcing the air particles closer together. Eventually then as condensation continues, the nebulous mass rises in temperature to visibility, forms a low temperature red N or M type star of enormous diameter and very low density, a red "giant" star. Continuing contraction is accompanied by increasing temperature and density and decreasing size, and the star passes through the yellow and white to the blue stage, through spectral types K, G and F to A, from temperatures of about 3,000° to 12,000° C. For the majority of the stars the turning point is about here and temperatures begin to fall. However, a few of the most massive of the stars still further increase in temperature to the B and O types at about 20,000° C.

The density has now so much increased that contraction can no longer supply sufficient heat to maintain the outflow of radiation and the temperature begins to fall, passing through the stages given above in the reverse order, through B, A, F, G, K, to M and N, through blue, white and yellow to low temperature, small diameter, high density red stars, "dwarfs," to extinction. It is evident that this second stage of the evolutionary process is common to the two theories. No account is taken in this brief sketch of the new generally recognized fact that contraction alone cannot supply more than a fraction of the energy given out as radiation. Whatever makes up the deficiency, however, is not likely to change the order above outlined but only markedly extend the time scale.

It is evident on this theory that the stars pass, for example, the M stage twice. First, shortly after condensation begins,

the star is an enormous spherical mass of very tenuous gas of a density of the order of a thousandth that of the air, while the diameter is measured then in the hundreds of millions of miles. The star is then literally and correctly called a giant M-type star. After passing through the A or B type, high temperature stage, it eventually reaches the M stage again, though now with a diameter probably considerably smaller than the Sun and a density much higher. It is now a "dwarf" M star and approaching the extinction stage. On the older theory, however, the M stage was only reached when the star was very contracted in diameter and very dense, a "dwarf." It is evident, therefore, that a knowledge of the dimensions of the stars will form a crucial test between the two theories and if it can be shown that there are both "giant" and "dwarf" stars of the same type and temperature, in the sky, it will be strong evidence of the substantial correctness of the new theory. Undoubtedly much of the knowledge of the dimensions of the stars we now possess has been due to methods developed by Russell and others as tests of the new hypothesis. This will be increasingly evident as we proceed with our main purpose, the determination of the dimensions of the stars, and it will be of interest to keep this thought in mind.

The obvious dimension of a star to be determined is the linear diameter, but a little reflection will show that the diameter by no means completely defines the conditions. It is readily conceivable that two stars might have the same diameter but be markedly different otherwise. One might be composed of very tenuous gas and the other liquid or solid. So that another dimension is required which may be either the mass or density. As the density is simply the mass per unit volume, grammes per cubic centimeter or pounds per cubic foot, it is evident that the density readily follows if mass and diameter are given, or if any two of the three factors, diameter, mass or density, are given the third is readily determinable. For example, if the diameter and mass are given, the density is obtained by dividing the mass by the volume, or similarly if the mass and density are given, the volume and consequently the diameter can be obtained by dividing the mass by the density.

Some methods of determining dimensions obtain the masses of the stars, others the densities, and still others the diameters. It is only when the methods can be so combined that two of these factors can be determined for the same star, that the dimensions can be completely obtained. The most obvious, to the layman at any rate, of the dimensions to be determined is the diameter, but curiously enough that has been the last, as well as probably the most difficult, to be obtained. The first determined was the mass and this may be considered the fundâmental dimension, as it is the only one that remains constant throughout the life history of a star for diameter and density, as we have seen, are continually changing.

It would seem impossible at first thought to determine the mass of any star, for all of them except our Sun are so far away that no telescope can show a sensible disc. Hence we can have no direct measure of its size or density, yet the method of weighing a star is essentially simple. It depends upon measuring the force which a star exerts by virtue of its mass, the force which is directly proportional to its mass, the universal force of gravitation. Everyone has heard of the law of gravitation—the attractive force between any two bodies in space is directly proportional to the product of the masses and inversely to the square of the distance between them. If then we know the force and the distance the masses can be determined. The force can only be determined when there is some other body on which the effects of the force can be observed, in other words when the star is attended by a revolving companion, when it is a double or, more correctly, a binary star. Only in the case of a binary star, a double star in which the components revolve around one another, whether this be a visual, spectroscopic or eclipsing binary, can the effects of the force be measured and the mass determined. A measure of the force and hence of the mass is obtained by a relation between the period of revolution and distance of the bodies and is a direct result of the law of gravitation. This relation, generally called the harmonic law, is expressed in the following terms:—The combined mass of one revolving system is to the combined mass of a second system as

the cube of the distance divided by the square of the period of the first system is to the cube of the distance divided by the square of the period of the second system. This relation, though not complex, can be simplified if we take the Earth-Sun system as the one to which the binary is to be compared. The mass of the Earth is only one three hundred and thirty thousandths of the Sun and so can be neglected in such calculations. If then we take the separation of the binary system in terms of the distance of the Earth from the Sun and the period in years, the above relation reduces to the following simple rule: The mass of any binary system is the separation cubed divided by the period squared times the mass of the Sun. Take, for example, first, the well-known star *Sirius,* which is accompanied by a 9th magnitude companion, the pair revolving around one another at 20 times the distance of the Earth from the Sun in a period of 49.3 years. The simple calculation for this and two other exceptional stars is given in the table.

MASSES OF BINARY STARS.

| | | |
|---|---|---|
| *Sirius* . . . . . . . . | Distance = 20. <br> Period = 49.3 | Mass $= \frac{20^3}{49.3^2} = 3.3\ \odot$ |
| Krueger 60 . . . . . | Distance = 11.1 <br> Period = 54.9 | Mass $= \frac{11.1^3}{54.9^2} = 0.45\odot$ |
| Y *Cygni* . . . . . . | Distance = .129 <br> Period = .0082 | Mass $= \frac{.129^3}{.0082^2} = 31.9\ \odot$ |

The method of mass determination above described applies only to those visual binaries, double stars, whose separation and orientation can be visually measured by the telescope, whose orbits have been determined and consequently whose periods and separations are known. Unfortunately the separation is given in seconds of arc and this can only be converted into miles to compare with the Earth-Sun distance when the parallax or distance is known. Although the orbits of nearly one hundred visual binaries have been computed, only a small proportion of these have reliable parallaxes and consequently the mass of about twenty only are accurately known. Aitken gives a list of fourteen in his book, "The Binary Stars," whose mean mass is 1.76 times the Sun, varying between 0.45 for Krueger 60 to 3.3 times the Sun for *Sirius.* These stars are nearly all of advanced spectral type and so far as it goes it indicates that the mass of the

individual stars of such binary systems is about the same as the Sun. Prof. Russell in support of his theory of stellar evolution has by an indirect statistical method found the average mass of about 350 binaries. The giants give values between 7 and 13 times the mass of the Sun, while the dwarfs range between 0.4 and 5.4 times the Sun's mass. That the values determined directly are smaller is due to the method of selection by which only the nearer stars have their parallax determined. We may say then that the average mass of the components of visual binaries of the dwarf class is not much different from the Sun.

Spectroscopic binary stars, those in which the separation is too small to be resolvable in any telescope and in which the duplicity is discovered by the variable radial velocity determined by the spectroscope, also lend themselves to the determination of the masses in certain cases. From measures of the radial velocity at different times it is possible to determine the character of the orbit, the eccentricity, the period and the projected separation. But the inclination of the plane of the orbit is indeterminable and consequently instead of the actual separation we know only its projection, its product by the sine of the inclination. When only one of the spectra can be seen and measured we obtain a function of the masses of the two components which does not give very definite information. But if both spectra can be measured we can determine the mass of each component, by the same method used for visual binaries. As only the projected and not the actual distance is known, these masses are always multiplied by the cube of the sine of the angle of inclination, $M \sin^3 i$. The values obtained are hence the minimum masses, while the actual masses may be considerably greater, the average value being about 50 per cent greater. Ludendorff has recently made an elaborate investigation of the masses of spectroscopic binaries and finds that the average minimum mass of 9 B-type stars is 10.8 times the Sun; of 17 A-type stars is 2.8 times the Sun and of 6 F-type stars is 1.9 times the Sun. The actual masses would probably be on the average 50 per cent greater than these values.

The masses of eclipsing binaries, which are spectroscopic

binaries whose plane of revolution is so nearly in the direction of the Earth that they mutually eclipse each other every revolution, can evidently be similarly determined. They have this advantage over the ordinary spectroscopic binary that the angle of inclination is known and consequently the actual masses are obtained. They range between 1.2 and 39 times the Sun's mass. The average mass of the 8 B-type is 16 times and of the 6 A to G type is 3 times the Sun. If we compare the masses of visual, spectroscopic and eclipsing binaries we find that the giants among the visual are about 10 times and among the spectroscopic and eclipsing about 16 times the Sun's mass, while the dwarfs average about 1.8 times the mass of the Sun in the directly determined visual binaries and about 2.8 times the Sun in the other classes.

AVERAGE MASSES OF BINARY STARS

| CLASS | METHOD | GIANTS | DWARFS |
|---|---|---|---|
| Visual | Actual | | 1.76 |
| Visual | Probable | 10.0 | 2.9 |
| Spectroscopic | Probable | 16.0 | 2.6 |
| Eclipsing | Actual | 16.0 | 3.0 |

This difference is readily explainable by the method of selection of spectroscopic and eclipsing binaries which are chosen for their brightness and large velocity displacement, both these factors indicating larger mass than average. It is probably safe to say that except for the relatively few giant stars, the average mass of the single component of any binary star is not much greater than the Sun. The smallest known mass is the faint component of Krueger 60, which is about 0.15 times the Sun, although theoretical considerations make it probable that the mass of Barnard's runaway star is only about one-fortieth that of the Sun, while the largest is V *Puppis,* 19.4 times the Sun. The bright component of Boss 6142, however, has a minimum mass of 18.5 times the Sun, which may be actually 25 or more times the Sun. The total range of mass then is only about a hundredfold.

It must not be forgotten, however, that these masses can only be determined from double stars, and we cannot say that the masses of isolated stars are similar. However, as the only single

star whose mass is known, the Sun, is of nearly the same mass as the average of the individual components of binary stars, it is probable the values are reliable for single as well as double stars. Moreover, dynamical considerations render it probable that the limiting masses do not greatly differ from those determined above.

Having thus obtained definite values of the masses, we now come to the consideration of the other dimensions, diameter and density of the stars. They offer quite a different problem in that while the mass remains constant throughout the life history of the star, the diameter and density certainly vary, as we will later see, through very wide limits. If the mass is known, and either diameter or density can be obtained, the other immediately follows. It will hence only be necessary to consider one of these factors, and I have chosen diameter, not only because it is the one that is directly determined, density being a derived factor, but also because it is the one which appeals to the layman who, if he is interested at all in the dimensions of the stars, will want to know their diameter rather than either their mass or density.

The problem of determining the diameters of the stars is by no means a simple one and has generally to be attacked indirectly. In order to prevent confusion of thought we must differentiate between the apparent and real diameters of the stars. The apparent diameter of any heavenly body is the angle which it subtends at the eye. Thus we can say that the apparent diameters of the Sun and Moon are nearly the same, about half a degree, while as everyone knows their real diameters are vastly different, 865,000 as compared with 2,180 miles. Although the planets appear as points without sensible diameter to the unaided eye, the telescope shows a measurable disc so that the apparent diameters of the Sun, Moon and planets can be easily measured, and when we know their distance, their actual diameter readily follows. But when we come to the stars it is quite a different matter.

My experience of some years in explaining to visitors what should be seen in a telescope has convinced me that the majority are surprised and disappointed at not seeing a magnified disc

when the telescope is pointed to a star. They do not realize the minuteness of the apparent diameters owing to the immense distances of these bodies and the consequent impossibility of even detecting, let alone measuring, the disc of any star with even the largest of telescopes. The hopelessness of this will be better realized when it is known that the image of a point source, and all the stars are practically points, at the focus of any telescope consists of a small central bright disc of light surrounded by dark and bright rings. This disc and ring system is due to interference of the waves of light, and although the central disc remains of the same linear diameter for all telescopes of the same aperture ratio, its angular diameter diminishes with the size of the objective. For the 100-inch telescope its diameter is about one-tenth of a second of arc, which at the Cassegrain focus would be slightly less than one-thousandth of an inch. Owing to the presence of the rings and of atmospheric tremor, the star would evidently require an apparent diameter considerably greater than one-tenth of a second, probably one-sixth or one-seventh of a second, to show a sensible disc. It is now fairly well established that the maximum apparent diameter of any star is about one-twentieth of a second, so that using the same proportion as above it would require an aperture of something like 25 or 30 feet to recognize and measure the apparent diameter of any star. Except then for the interferometer method, which will be later referred to, the diameters of stars must be determined by indirect methods.

The most positive and accurate of these indirect measures is obtained, just as in the determination of masses, by the study of double stars, indeed of one particular class of double stars—the eclipsing binaries or variables. It will be remembered that we obtained the most accurate determinations of mass from eclipsing binaries, so also from eclipsing binaries can we obtain the most accurate determinations of diameter and density. As has been previously shown, a knowledge of the diameters and densities of stars will form a critical test between the rival theories of stellar evolution. Prof. Russell, who introduced the newer theory, was probably thus induced to develop a complete and

beautiful method of discussing the light curves of eclipsing variables. The light curve of an eclipsing variable is formed by plotting the change in the light with the time due to the mutual eclipses of the components, and Prof. Russell was able by his method to determine from the light curve the shape and inclination of the orbit of revolution, the shape of the two stars and their relative sizes as compared with their distance apart. Using this method with probably the same end in view, Dr. Harlow Shapley, a student and associate of Prof. Russell's, determined the photometric orbits of 90 eclipsing variables, all of those bodies which had sufficiently well determined light curves.

Only the relative dimensions of the component stars as compared with their separation are thus obtained, and in order to get some idea of the average diameters, Shapley assumed that the mass of the components was equal and each the same as the Sun. It is then easy from the harmonic law to determine the separation and hence the diameters and densities of these 90 systems. Shapley has tabulated these values and finds for the diameters a range between 0.6 and 110 times the Sun, although one uncertain case gave a diameter of 700 times the Sun. The values for the densities to which, however, a further correction was applied, ranged from less than one-millionth to over five times the Sun's density. While these results are possibly not far out on the average, they are based on assumptions which are not universally true and although they served their purpose in the evolutionary theory by unmistakably showing the presence of both giant and dwarf stars, it is of great interest and value to get actual dimensions.

Just as in the determination of mass, absolute values could only be obtained from eclipsing variables when the spectra of both components were measurable, so in the determination of diameters, actual linear values can only be obtained when both spectra are measurable. From the spectroscopic orbit, as previously stated, we obtain only the projected length of the separation and in order to get the actual separation we must know the angle of inclination. The photometric orbit gives this angle and the ratio of the diameters to the separation, so their determination is a mere matter of multiplication. To summarize:—

The spectroscopic orbit gives the separation multiplied by the sine of the inclination, but as this angle is given by the photometric orbit we get the actual separation of the two stars. Then from the photometric orbit we have the ratio of the diameters to the separation, hence the actual diameters of the two stars.

Of the 90 eclipsing variables for which Shapley obtained photometric orbits only 14 with doubled spectra have had their spectroscopic orbits determined. The speaker, at the Dominion Astrophysical Observatory, has determined 7 of these orbits, while 7 have been obtained elsewhere. The table gives the spectrum, diameters, masses and densities of these 14 stars.

Absolute Dimensions of Eclipsing Variables
*In Terms of Sun*

| STAR | TYPE | DIAMETERS $d_1$ | $d_2$ | MASSES $m_1$ | $m_2$ | DENSITIES 1 | 2 |
|---|---|---|---|---|---|---|---|
| β *Aurigæ* . . . . . . | Ap | 2.8 | 2.8 | 2.4 | 2.4 | 0.11 | 0.11 |
| u *Herculis* . . . . . | B3 | 4.6 | 5.3 | 7.7 | 2.9 | 0.095 | 0.022 |
| V *Puppis* . . . . | B1 B3 | 8.4 | 7.7 | 19.4 | 19.4 | 0.042 | 0.055 |
| β *Lyræ* . . . . . | B8 B5 | 16.2 | 40.6 | 1.4 | 14.2 | 0.0006 | 0.0004 |
| RX *Herculis* . . . . | B9 | 1.5 | 1.4 | 0.9 | 0.9 | 0.25 | 0.34 |
| W *Urs. Maj.* . . . . | G | 0.8 | 0.8 | 0.7 | 0.5 | 2.8 | 1.9 |
| Z *Herculis* . . . . . . | F | 1.8 | 3.3 | 1.6 | 1.3 | 0.3 | 0.04 |
| U *Ophiuchi* . . . . | B5 | 3.2 | 3.2 | 5.4 | 4.7 | 0.18 | 0.16 |
| RS *Vulpec.* . . | B8 B9 | 2.0 | 10.2 | 5.4 | 1.7 | 0.63 | 0.0016 |
| U *Coronæ* . . . . . | B3 | 2.0 | 4.7 | 4.3 | 1.6 | 0.175 | 0.015 |
| TX *Herculis* . . . . | A2 | 1.3 | 1.3 | 2.0 | 1.8 | 0.87 | 0.75 |
| Y *Cygni* . . . . . . | B2 | 4.6 | 4.6 | 16.6 | 15.3 | 0.170 | 0.158 |
| Z *Vulpec.* . . . . . . . | B3 | 4.2 | 4.5 | 5.2 | 2.4 | 0.085 | 0.033 |
| TV *Cass.* . . . . . . | A | 2.7 | 2.3 | 2.0 | 1.2 | 0.010 | 0.010 |

The diameters range between 0.78 and 40 times the Sun and the densities between 0.0004 and 2.8 times the Sun. These absolute measured dimensions are sufficient to show the presence of giants and dwarfs and to substantiate Shapley's theoretical results of several times greater range.

Again, however, these conclusions as to diameter and density, as well as the previous ones in regard to mass, are derived from double stars, and we have not yet found the dimensions of any isolated star except our Sun. Although it seems unlikely that the dimensions of single stars should be markedly different from those of double stars, some independent evidence is desirable.

Although an independent method of obtaining the diameters of the stars has recently been developed, the measure of the

apparent diameter by the interferometer, the actual measures of diameter were preceded by a theoretical discussion, also by Prof. Russell, who, I venture to say, was probably also in this case influenced by its bearing on his evolutionary hypothesis. The apparent brightness of any star evidently depends upon its area and its surface brightness, the brightness per unit area, and is proportional to the product of the two. Hence for all stars of the same surface brightness, the area is proportional to the apparent brightness and hence the apparent diameter proportional to the square root of the apparent brightness. For example, the star *Capella*[2] is of the same spectral type and probably also of the same surface brightness as the Sun. We know that the Sun is nearly 27 magnitudes brighter than *Capella* or about 60,000,000,000 times. Hence the Sun should have an apparent diameter the square root of this quantity times greater than *Capella* or the apparent diameter of the Sun should be about 240,000 times the apparent diameter of *Capella*. The apparent diameter of the Sun is about 32 minutes, 1,920 seconds, so that the apparent diameter of *Capella* will be 1,920 divided by 240,000, or about eight one-thousandths of a second of arc. Evidently then it is easy to calculate from the apparent magnitude the apparent diameter of any star of the same spectral type as the Sun. Further, the change of surface brightness with change of spectral type or color can be approximately determined and we can evidently apply this ratio to the determination of the diameters of stars of other spectral types, other colors, than the Sun. Prof. Russell has thus calculated the apparent diameter of all the brightest stars in the sky, and his values were given in the A. S. P. for December, 1920. Prof. Eddington about the same time independently calculated apparent diameters and found values somewhat higher than Russell.

It would evidently be of the greatest value in order to test the correctness of this theoretical work to have some direct measure of the apparent diameters of stars. Although, as we have already found, it is hopeless to even see, let alone measure, stellar discs in any telescope, the genius of Prof. Michelson, combined

[2]*Capella* is spectroscopically double and these calculations assume all the light of the system collected into one G-type star.

with the ability of the staff and the completeness of the instrumental resources of Mount Wilson, have enabled the diameter of three stars to be measured by an interferometer attached to the 100-inch telescope. As this great advance has already been fully described in technical and other journals, I will not attempt to describe it. The measured diameters come about midway between the calculated values of Russell and Eddington, so that we can have confidence in the substantial accuracy of the computed values.

Both the theoretical and observational methods give, however, only the apparent diameters of the stars, the angle which their discs subtend at the eye or the interferometer. To determine the linear diameter in miles it is necessary to know the distance or parallax. The parallax is the angle subtended at the star by the distance of the Earth from the Sun, hence the ratio of the linear diameter of the star to this distance is given by the ratio of the apparent diameter to the parallax. The apparent diameter of *α Orionis* is $0''.045$, while its parallax may be taken as $0''.018$, hence its diameter is 45/18 of 93,000,000, or about 235,000,000 miles.

There have only been announced, to the present, the measured diameters of *Betelgeuse, Antares* and *Arcturus,* but we have the theoretically computed apparent diameters of any star by Russell's method, which as we have seen gives slightly smaller diameters than the measured This difference is probably due to an incorrect estimate of the relative surface brightness of stars of spectral types or colors different from the Sun. I have accordingly taken the liberty of applying the probable correction, deduced from the measured values, to Russell's figures and, applying the most probable parallax, have computed the apparent and actual diameters, the apparent and real brightness, and the density, using reasonable average values of the mass, when that is not directly known, of some of the brightest and faintest stars. These figures should not be considered by any means as final, but are probably of the right order and serve to give us ideas of

the dimensions of the exceptional stars. That of the average star is more likely nearly the same order as the Sun.

COMPUTED DIMENSIONS OF TYPICAL STARS

| STAR | TYPE | APP. MAG. | APP. DIAM. | PARAL-LAX | ASS'D MASS | DENSITY | BRIGHT-NESS | DIAMETER |
|---|---|---|---|---|---|---|---|---|
| *Betelgeuse* . . | Ma | 0.9 | .044 | .018 | 30 | .0000012 | 1450 | 235,000,000 |
| *Antares* . . . | Map | 1.2 | .038 | .013 | 30 | .0000010 | 1600 | 275,000,000 |
| *α Herculis* . . | Mb | 3.5 | .015 | .007 | 30 | .0000020 | 710 | 200,000,000 |
| *Aldebaran* . . | K5 | 1.5 | .027 | .075 | 10 | .00017 | 36 | 33,000,000 |
| *Arcturus* . . . | K0 | 0.2 | .023 | .095 | 10 | .0007 | 78 | 23,000,000 |
| *Capella** . . . | G0 | 0.2 | .0082 | .071 | 4.6 | .006 | 78 | 8,000,000 |
| *Pollux* . . . . | K0 | 1.2 | .015 | .095 | 5 | .0012 | 31 | 13,000,000 |
| *Procyon* . . . | F5 | 0.5 | .0048 | .328 | 2 | .60 | 5 | 1,400,000 |
| *Sirius* . . . . . | A0 | —1.6 | .0057 | .376 | 2.5 | .62 | 26 | 1,430,000 |
| *Vega* . . . . . | A0 | 0.1 | .0026 | .094 | 5 | .21 | 86 | 2,600,000 |
| *Rigel* . . . . . | B8 | 0.3 | .0019 | .007 | 30 | .0012 | 13500 | 25,000,000 |
| Krueger 60 . | Mb | 9.3 | .0011 | .260 | .42 | 4.0 | .002 | 360,000 |
| Barnard's Star | Mb | 9.7 | .0009 | .53 | .023 | 4.0 | .0004 | 155,000 |
| *Prox. Cent.* . | N | 11.0 | .0017 | .76 | .055 | 4.0 | .00006 | 207,000 |

**Capella* is spectroscopically double and the dimensions are calculated on the assumption that all the light is collected into one G-type star.

We readily see from the dimensions of the stars thus obtained, the enormous diameters, 300 times the Sun, and low densities, about one-millionth of the Sun, one-thousandth of the atmosphere, of the red giants as compared with the small diameters, one-fifth the Sun, and high densities of the red dwarfs, that there appears to be no escape from the conclusion that the newer idea of stellar evolution, which requires dimensions of these orders, is the most probable.

There are not many episodes in the history of astronomy more interesting than the development and substantiation of the now generally accepted theory of the evolution of the stars. Nor do I know of any which illustrate more clearly the value of carefully planned researches towards a definite end. Not only have these resulted in clearer conceptions of the order and beauty of the universe but our knowledge of the dimensions of the stars has been enormously increased.